Name of College - J-Bag

Dept - Mathematics

Topic - Problem based on the radius of curvature for the polar curve (Differential Calculus) Contd

Class - B.Sc (HONS)

Time - 10:15 to 11:00 A.M

11:00 A.M to 11:45 A.M

Date - 31-02-2020

By Samrendra Kumar

Problem: Find the radius of curvature at any point of the curve.

$$r^m = a^m \cos m\theta.$$

Solution → By the question

Equation of curve is

$$r^m = a^m \cos m\theta$$

Taking logarithm of both sides

$$m \log r = m \log a + \log \cos m\theta.$$

Diff. with respect to θ.

$$m \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = \frac{1}{\cos m\theta} (-\sin m\theta)(m)$$

$$\Rightarrow \frac{m}{r} \frac{dr}{d\theta} = -\tan m\theta \times m$$

$$\Rightarrow \frac{1}{r} \frac{dr}{d\theta} = -\tan m\theta$$

$$\Rightarrow r \cdot \frac{d\theta}{dr} = -\cot m\theta = \tan\left(\frac{\pi}{2} + m\theta\right)$$

But $r\frac{d\theta}{dr} = \tan\phi = \tan\left(\frac{\pi}{2} + m\theta\right)$

$\Rightarrow \phi = \frac{\pi}{2} + m\theta$

Also $p = r\sin\phi$

$= r\sin\left(\frac{\pi}{2} + m\theta\right)$

$= r\cos m\theta$

But $r^m = a^m \cos m\theta$

$\Rightarrow \cos m\theta = \frac{r^m}{a^m}$

$\Rightarrow p = r \cdot \frac{r^m}{a^m}$

$\Rightarrow a^m p = r^{m+1}$

Differentiating with respect to r

we have $a^m \frac{dp}{dr} = (m+1) r^m$

$\Rightarrow \frac{dr}{dp} = \frac{a^m}{(m+1) r^m}$

$\therefore \ell = r \cdot \frac{dr}{dp}$

$= r \cdot \frac{a^m}{(m+1) r^m}$

$= \frac{a^m}{(m+1) r^{m-1}}$

Problem 2

If $r = a\sec 2\theta$ show that

$$\ell = -\frac{r^4}{3p^3}$$

for we ~~need~~ have to reduce the equation $r = a\sec 2\theta$ in the pedal form

Taking log of both sides.

$$\log r = \log a + \log \sec 2\theta.$$

Diff. with respect to θ.

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{\sec 2\theta} \times \sec 2\theta \tan 2\theta \times 2$$

$$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = 2\tan 2\theta$$

$$\Rightarrow r\frac{d\theta}{dr} = \frac{1}{2\tan\theta} \qquad \text{But } r\frac{d\theta}{dr} = \tan\varphi$$

$$\Rightarrow \tan\varphi = \frac{1}{2\tan\theta}$$

Again $p = r\sin\varphi$

$$= r\frac{1}{\sqrt{1+4\tan^2 2\theta}} \qquad \frac{r}{\sqrt{1+4\tan^2 2\theta}}.$$

$$= r\frac{1}{\sqrt{4\sec^2 2\theta - 3}} \qquad \because r = a\sec 2\theta$$

$$\Rightarrow \frac{r}{a} = \sec 2\theta$$

$$\frac{r^2}{a^2} = \sec^2 2\theta.$$

$$\Rightarrow p^2 = \frac{r^2}{\cancel{4\sec^2 2\theta - 3}\ 4\sec^2 2\theta - 3}$$

$$= \frac{r^2}{4\left(\frac{r^2}{a^2}\right) - 3}$$

$\Rightarrow p^2 = \dfrac{r^2 a^2}{4r^2 - 3a^2}$

$\Rightarrow p^2(4r^2 - 3a^2) = r^2a^2$

$\Rightarrow 4r^2p^2 - 3a^2p^2 = r^2a^2$

$\Rightarrow r^2(4p^2 - a^2) = 3a^2p^2$

Now diff. w.r.t. respect to p, we get

$r^2(8p) + (4p^2 - a^2)\,2r\dfrac{dr}{dp} = 6a^2p$

$\Rightarrow 4r^2p + (4p^2 - a^2)\,r\dfrac{dr}{dp} = 3a^2p$

$\Rightarrow (4p^2 - a^2)\rho = p(3a^2 - 4r^2)$

$\Rightarrow \dfrac{3a^2p^2}{r^2}\rho = p\left(\dfrac{-r^2a^2}{p^2}\right)$

$\therefore \rho = -\dfrac{r^4}{3p^3}$

[Since $p^2(4r^2 - 3a^2) = r^2a^2$

and $r^2(4p^2 - a^2) = 3a^2p^2$]

Find the radius of curvature at any point on the curve

$r = ae^{\theta\cot\alpha}$

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Solution :→

We have

$r = ae^{\theta\cot\alpha}$

$\therefore \frac{dr}{d\theta} = ae^{\theta\cot\alpha} \cdot \cot\alpha = a\cot\alpha\, e^{\theta\cot\alpha}$

$= \cot\alpha(ae^{\theta\cot\alpha})$

$= r\cot\alpha$

$\Rightarrow \frac{d\theta}{dr} = \frac{1}{r\cot\alpha}$

Since $\tan\phi = r\frac{d\theta}{dr} = r \cdot \frac{1}{r\cot\alpha} = \tan\alpha \quad \therefore \phi = \alpha$

Since $p = r\sin\phi$

$= r\sin\alpha$

$\therefore \frac{dp}{dr} = \sin\alpha$

$\Rightarrow \frac{dr}{dp} = \frac{1}{\sin\alpha}$

$\therefore \rho = r\frac{dr}{dp} = \frac{r}{\sin\alpha} = \underline{r\operatorname{cosec}\alpha}.$

Find the radius of curvature for the circle $r = a\sin\theta$

here $r = a\sin\theta$

$\frac{dr}{d\theta} = a\cos\theta \qquad \frac{d^2r}{d\theta^2} = -a\sin\theta.$

we have $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

$$= \frac{(a^2\sin^2\theta + a^2\cos^2\theta)^{3/2}}{a^2\sin^2\theta + 2a^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{(a^2)^{3/2}(\sin^2\theta + \cos^2\theta)^{3/2}}{2a^2} = \frac{a^3}{2a^2} = \frac{a}{2}$$

$$\therefore \rho = \frac{a}{2}$$

<u>Problem</u> :→ for any Curve Prove the formula

$$\rho = \frac{r}{\sin\varphi\left(1+\frac{d\varphi}{d\theta}\right)} \quad \text{Where } \tan\varphi = r\frac{d\theta}{dr}$$

<u>Solution</u>

Since we know

$$\sin\varphi = r\frac{d\theta}{ds} \qquad \rho = \frac{ds}{d\psi}$$

and $\psi = \theta + \varphi$

Now $\sin\varphi\left(1+\frac{d\varphi}{d\theta}\right) = r\frac{d\theta}{ds}\left(1+\frac{d\varphi}{d\theta}\right)$

$$= r\frac{d\theta}{ds} + r\frac{d\theta}{ds}\cdot\frac{d\varphi}{d\theta}$$

$$= r\frac{d\theta}{ds} + r\frac{d\varphi}{ds}$$

$$= r\left(\frac{d\theta}{ds} + \frac{d\varphi}{ds}\right)$$

$$\therefore \rho = \frac{r}{\sin\varphi\left(1+\frac{d\varphi}{d\theta}\right)} = r\frac{d}{ds}(\theta+\varphi) = r\frac{d\psi}{ds} = r\cdot\frac{1}{\rho}$$

proved